**SubSpaces**

A subset W of a vector space V is called a subspace of V if W is itself a vector space under the addition and scalar multiplication defined on V.

**Subspace Criteria:**

If W is the subset of a vector space V, then W is called subspace if and only if the following conditions hold:

1. (Closed under addition) If and are vectors in W, then is in W.
2. (Closed under scalar multiplication) If k is any scalar and is any vector in W then is in W.

Or in other words, W is subspace of V if and only if it is closed under addition and scalar multiplication.

**Example 1:** If V is any vector space and if is the subset of V that consists of the zero vector only, then W is closed under addition and scalar multiplication since

for any scalar k

We call W the zero subspace of V.

**Example 2:**

Prove and disprove W is subspace of

Solution:

1. W is closed under addition, because

Let,

1. W is not closed under scalar multiplication, because if

But

So W is not subspace of V.

**Example 3:**

Which of the following subset of with usual operations of addition and scalar multiplication are subspaces?



**Solution:**

1. is closed under addition but not closed under scalar multiplication, because
2. Done in example 2.

As in . To see whether is subspace , Let

is closed under addition.

Let , k is any scalar then

is closed under scalar multiplication.

is subspace of .

**Example 4:**

Which of the following subset of with usual operations of addition and scalar multiplication are subspaces?

1. All vectors of the form
2. All vectors of the form
3. All vectors of the form where
4. All vectors of the form where
5. All vectors of the form .

**Solution:**

1. is closed under addition and scalar multiplication as:
2. Let and k is any scalar, then

is subspace of .

Let’s check the first property, i.e. is closed under addition.

Let

is not subspace of .

1. is closed under addition and scalar multiplication as:

Then

So is closed under addition.

1. Let and k is any scalar, then

is closed under scalar multiplication.

So, is subspace.

1. All vectors of the form where

Then

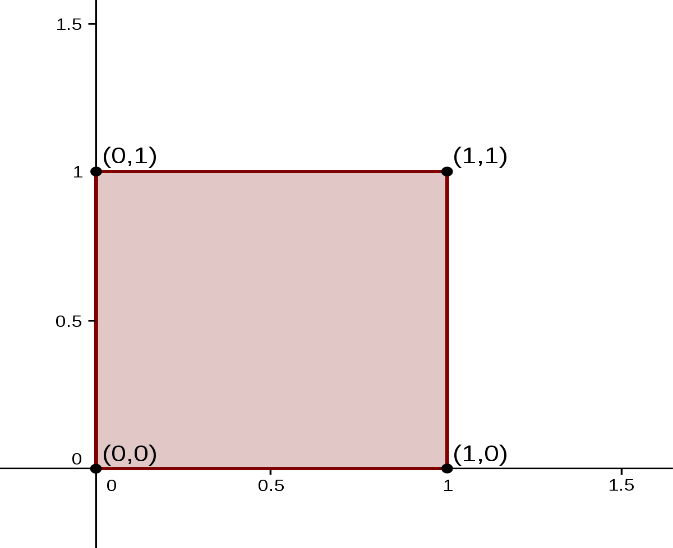
So is not closed under addition.

So is not subspace of

1. All vectors of the form .(Do it yourself)

**Example 5.**

Consider the unit square shown in the accompanying figure. Let W be the set of the form , where . That is W is the set of all vectors whose tail is at origin and whose head is a point inside or on the square. Is W a subspace of ? Explain.

**Solution:**

Let

Then

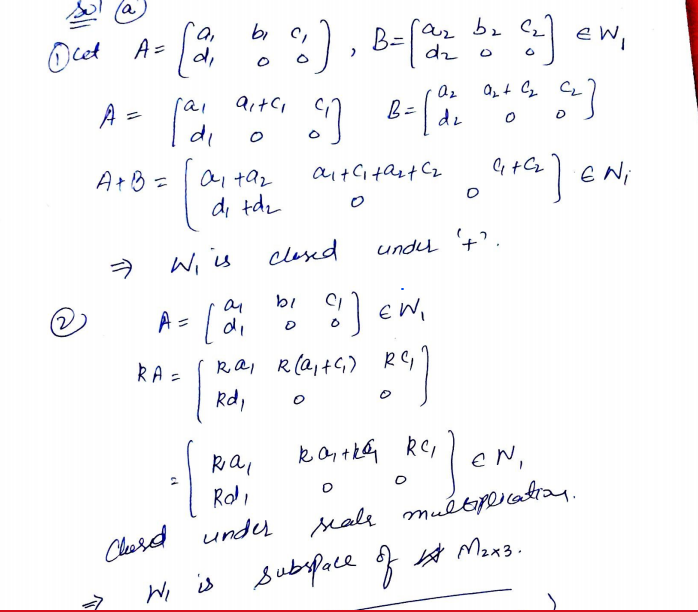
W is not closed under addition.

W is not subspace of

**Example 6.**

Which of the given subsets of the vector space of all matrices are subspace? The set of all matrices of the form?

Solution:



C: do it your self